

# Ellipses vs Hyperbolas (without worrying whether $a$ or $b$ goes under $x$ or $y$ )

The definitions, equations and properties for ellipses and hyperbolas are very similar but also very subtly different

|  | Ellipse   | Hyperbola  |
|--|---|--|
| <b>Basic equation<br/>when center at origin<br/>(use shifts<br/>when center not at origin)</b> | $\frac{x^2}{m^2} + \frac{y^2}{n^2} = 1$   | $\frac{x^2}{m^2} - \frac{y^2}{n^2} = 1$ if hyperbola opens horizontally<br>(coefficient of $x^2$ positive)<br>$\frac{y^2}{n^2} - \frac{x^2}{m^2} = 1$ if hyperbola opens vertically<br>(coefficient of $y^2$ positive) |
|  | To find the number under $x^2$ ,<br>start at the center of the ellipse or hyperbola,<br>and move horizontally until you hit a point on the conic.<br>Square that distance.<br><br>To find the number under $y^2$ ,<br>start at the center of the ellipse or hyperbola,<br>and move vertically until you hit a point on the conic.<br>Square that distance.<br><br>For a hyperbola, only one of the two above methods will work (*)<br>(for the “positive” variable – that does not have the subtraction in front of it),<br>since you won’t hit the hyperbola if you move in the other direction. |  |
| <b>Distances from<br/>any point to foci</b>  | add up to fixed constant  | differ by fixed constant   |
| <b>Location of vertices</b>  | 2 points which are farthest apart   | 2 points on 2 branches which are closest together  |
| <b>Equation regarding<br/>distance to foci</b>   | $PF_1 + PF_2 = \text{constant}$ (distance between vertices)   | $ PF_1 - PF_2  = \text{constant}$ (distance between vertices)  |
| <b>Axis through vertices</b>   | Major axis  | Transverse axis  |
| <b>Axis perpendicular to<br/>axis through vertices</b>   | Minor axis  | Conjugate axis   |
| <b>Location of foci</b>  | Along axis through vertices – foci lie between center and vertices<br>(conic wraps around foci)   | Along axis through vertices – vertices lie between center and foci<br>(conic wraps around foci)  |
| <b>Equation for distance<br/>from center to focus</b><br>$c^2 =$                               | difference of numbers under $x^2$ and $y^2$<br>(bigger number minus smaller number)   | sum of numbers under $x^2$ and $y^2$   |
| <b>Location of center</b>  | Midpoint of major axis (between vertices)<br>Midpoint of minor axis<br>Midpoint between foci<br>Intersection of major & minor axes  | Midpoint of transverse axis (between vertices)<br>Midpoint of conjugate axis<br>Midpoint between foci<br>Intersection of asymptotes, transverse & conjugate axes   |

The slopes of the asymptotes of a hyperbola are  $\pm \sqrt{\frac{\text{coefficient under } y^2}{\text{coefficient under } x^2}}$ . (Remember, slope =  $\frac{\Delta y}{\Delta x}$ .) Both asymptotes of a hyperbola pass through the center of the hyperbola.

(\*) For the number under the other variable, use the formula for the foci, or the formula for the slopes of the asymptotes.